1B: Propositions and Truth Values - SOLUTIONS

<u>Group Activity</u>

Have the reader in your group read each question and as a group, discuss your answer.

1. Discuss each statement and decide which are propositions. Circle or highlight them.
   a) Are you hungry?

   b) Janae was nominated for student council president.

   c) Four pounds less.

   d) 7 + 8 = 33

   e) I have two siblings.

2. A proposition is given, represented by the letter p. Write the wording for the negation and double negation. (Note: not p can also be represented in symbols by ~p.)

p: Hannah eats apples.

not p: **Hannah does not eat apples**

not not p: **Hanna refuses to not eat apples (you can be creative with this one)**

3. More Negations. Read the following and decide whether logging will continue. *"The House failed to overturn a veto on a bill that would stop logging."*

The bill would stop logging. The veto would continue logging. Overturning the veto would stop logging. Failing to overturn the veto continues logging.

4. Complete the truth table given the following propositions. Then explain what you ate for breakfast in each case.

p: I ate bacon for breakfast.   *q:* I ate eggs for breakfast.

p	*q*	*p **and** q*	*Explanation – What did you eat for breakfast?*
T	T	T	**I ate eggs and bacon**
T	F	F	**I ate bacon and not eggs**
F	T	F	**I ate eggs but not bacon**
F	F	F	**I ate neither eggs nor bacon**

5. Given the conditional statement, complete the truth table. Refer to your notes for help if needed. (Note: "if p, then q" can be written in symbols as p→q.)

"If I am elected, then I will reduce college tuition in Oregon."
p: I am elected   *q:* I will reduce college tuition in Oregon

p	*q*	*if p, then q* *p → q*	*Meaning – Have I kept my campaign promise?*
T	T	T	**I got elected and I reduced college tuition in Oregon. Yes, I kept my promise.**
T	F	F	**I got elected and I did not reduce college tuition in Oregon. No, I did not keep my promise.**
F	T	T	**I did not get elected and I reduced college tuition in Oregon. I did not promise anything about if I did not get elected so I have kept my promise and this is true.**
F	F	T	**I did not get elected and I did not reduce college tuition in Oregon. I did not promise anything about if I did not get elected so I have kept my promise and this is true.**

6. Pizza Truth Table

Let P represent pepperoni, H represent ham, and M represent mushrooms.

a. Complete each row by determining whether each statement is true or false. Don't worry about the meaning yet.

P	H	M	P and H	H or M	P and H and M	not P	not M	not P or not M
T	T	T	T	T	T	F	F	F
T	T	F	T	T	F	F	T	T
T	F	T	F	T	F	F	F	F
T	F	F	F	F	F	F	T	T
F	T	T	F	T	F	T	F	T
F	T	F	F	T	F	T	T	T
F	F	T	F	T	F	T	F	T
F	F	F	F	F	F	T	T	T

b. Now that you have the table filled in, Identify the row for each type of pizza given.
Pepperoni and Mushroom Pizza: Row 3
Cheese Pizza: Row **8**
Veggie Pizza (peppers, onions, mushrooms, olives): Row **7**
Supreme Pizza (ham, pepperoni, sausage, olives, mushrooms, onions, peppers): Row **1**

c. Explain the result (T or F), in the context of the pizza and its toppings.
  i.   Row 8 Column 9
Not P or not M is true so the pizza either does not have pepperoni or does not have mushrooms or does not have either. Row 8 is the cheese pizza so that makes sense.
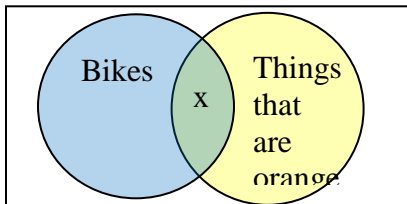
  ii.   Row 3 Column 4
P and H is false, so the pizza is missing either pepperoni or ham. Row 3 is the Pepperoni and Mushroom so it is missing the ham.

1C: Sets and Venn diagrams

7. Draw a Venn diagram for each categorical proposition. Then write on the line whether the sets are disjoint, overlapping, or one is a subset of the other.
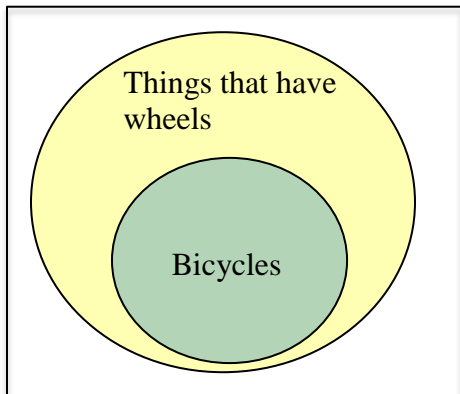
                                                                    Relationship between the Sets
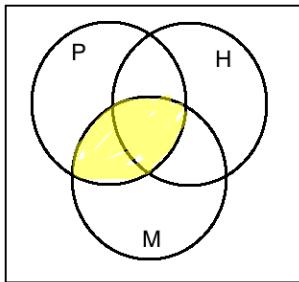
a. Some bikes are orange.   <u>**Overlapping**</u>

b. All bicycles have wheels.   <u>**Subset**</u>
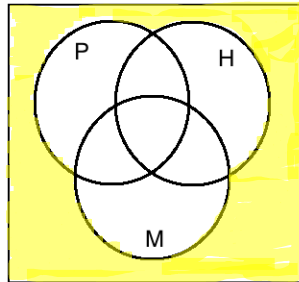
8. Pizza Venn Diagrams

Let's continue the last example with P representing pepperoni, H representing ham, and M representing mushrooms. Shade or color in the appropriate area(s) for each pizza on the Venn diagrams below.
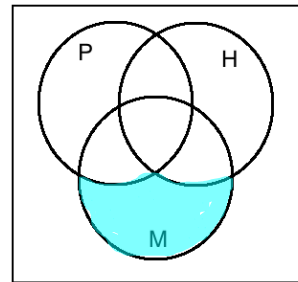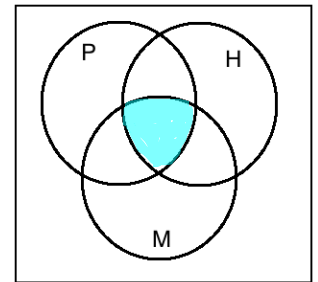
| Pepperoni & Mushroom	Cheese	Veggie	Supreme

9. Finding values for regions on a Venn diagram

150 people attending a concert were asked if they played piano, guitar, or drums.

10 could play all three.
73 could play guitar.
18 couldn't play any of these instruments.
21 could play piano and drums.
49 could play at least two of the instruments.
13 could play piano and guitar but not drums.
69 could play drums or guitar but not piano.

Let P represent piano,
    G represent guitar, and
    D represent drums.

Calculate and fill in the values for all eight regions.
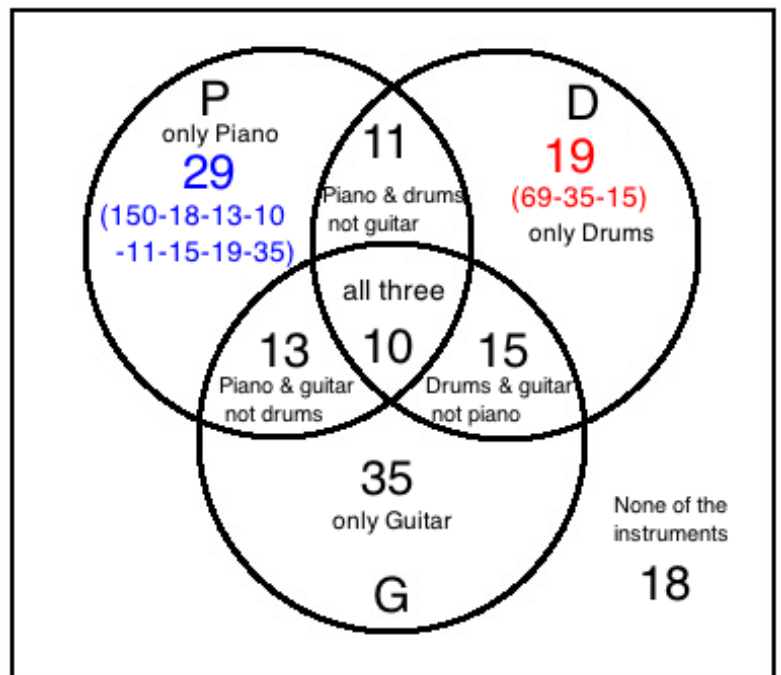
1 D: Analyzing Arguments

10. Determine whether each argument is inductive or deductive.

  a. All cats have a keen sense of smell. Fluffy is a cat, so Fluffy has a keen sense of smell.
     This argument goes from general to specific so it is deductive.

  b. All brown dogs in the park are small dogs. Therefore, all small dogs are brown.
   **This argument goes from specific to general so it is inductive.**

  c. My friends who are in college eat pizza. Pizza is affordable. Therefore, all college students eat pizza.
     This argument goes from specific to general so it is inductive.

11. Draw a Venn diagram for each deductive argument and determine whether it is valid and sound. (Problems adapted from David Lippman http://www.opentextbookstore.com/mathinsociety/index.html)

  a.   Premise: All cats are mammals (T)

        Premise: A tiger is a cat (T)

        Conclusion: A tiger is a mammal (T)

        The conclusion is valid/invalid
        The conclusion is sound/not sound
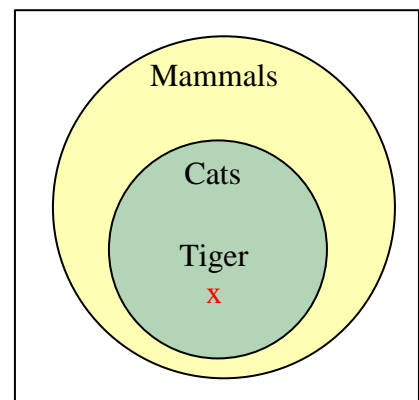
  b.   Premise: If you live in Portland, you live in Oregon (T)

        Premise: Cara does not live in Oregon (F)

        Conclusion: Cara does not live in Portland (F)

        The conclusion is valid/invalid
        The conclusion is sound/not sound
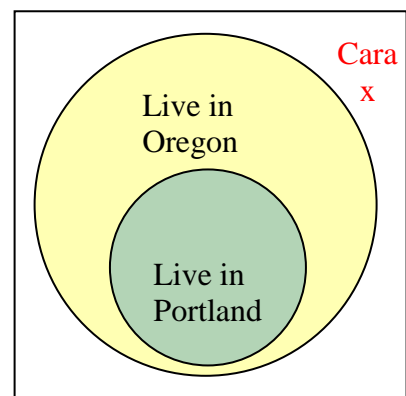
        The conclusion follows from the premises so, the argument is valid. The second premise is false, however, so the argument is not sound.

Here are a few more vocabulary words:

Related Statements	In Words	In Symbols
The original <u>conditional</u> is	"if p, then q"	$p \rightarrow q$
The <u>converse</u> is	"if q, then p"	$q \rightarrow p$
The <u>inverse</u> is	"if not p, then not q"	$\sim p \rightarrow \sim q$
The <u>contrapositive</u> is	"if not q, then not p"	$\sim q \rightarrow \sim p$

12. Here is a conditional statement: If you have finished your homework, then you get ice cream. Identify p and q to help you write each statement below.

    a. Write the converse of the original statement.
    If q, then p
    If you get ice cream, then you have finished your homework.

    b. Write the inverse of the original statement.
    If not p, then not q
    If you have not finished your homework, then you did not get ice cream.

    c. Write the contrapositive of the original statement.
    If not q, then not p
    If you have not gotten ice cream, then you have not finished your homework.

    d. If the original statement is true, which of the other three are also true?
    The contrapositive is true.

1A: Logical Fallacies

A **logical fallacy** is a type of flawed or misleading argument.

We are going to study ten common fallacies (but there are many more that you can find on the internet).

The ten fallacies are listed on the yellow and red cards (like fútbol or soccer penalty cards) with their descriptions on the back. There are ten examples on white cards.

10 Common Logical Fallacies

13. Using the cards provided, match the example with the description and write the name of the fallacy.

Fallacy	Description	Examples
Appeal to Popularity	Many people believe p is true; therefore, p is true.	"Everyone is buying these jeans, so they are the best."
False Cause or Post Hoc	A came before B; therefore, A caused B.	"Today I wore a red shirt, and my team won! I will wear a red shirt every time they play to make sure they keep winning."
Appeal to Ignorance	There is no proof that p is true; therefore, p is false.	"No one has proven it isn't Bigfoot in the photo, so it must be Bigfoot."
Hasty Generalization or Correlation Implies Causation	A and B are linked one or a few times; therefore, A causes B or B causes A.	"Months with high ice cream sales also have a high rate of deaths by drowning. Therefore, ice cream must be causing people to drown."
Limited Choice or False Dilemma	p is false; Therefore, only q can be true.	"Either those lights in the sky were an airplane or aliens. There are no airplanes scheduled for tonight, so it must be aliens."
Appeal to Emotion	p is associated with a positive emotional response; therefore, p is true.	"If you care enough to send the very best you will buy this brand of card."
Circular Reasoning	p is true; therefore, p restated in a different way is true.	"Only an untrustworthy person would run for office. The fact that politicians are untrustworthy is proof of this."
Diversion or Red Herring	p is related to q and I have an argument regarding q; therefore, p is true.	"We can't worry about the environment, we're in the middle of a war."
Straw Person	I have an argument regarding an oversimplified or distorted version of p; therefore, I hope you will believe it is true about the real version of p.	"Senator Jones has proposed reducing military funding by 10%. Apparently, he wants to leave us defenseless against attacks by terrorists."
Personal Attack or Ad Hominem	I have a problem with the person or group claiming p; therefore, p is not true.	"Vote against the healthcare bill because 'Lying Laura' supports it."

More Practice
These problems can be used to check your understanding, help with online homework and study for tests. The answers can be found on my website.

1B Truth Values

1. The following proposition has the form p and q. Decide whether each part is true or false and whether the entire proposition is true or false.

$25+30=55$ *and* $55 \div 11 = 6$ **p is true but q is false. The statement p and q is false.**

2. Complete the truth table.

r	s	t	r and t	s or t	not (s or t)	If s then (r and t)	not r	If s then not r
T	T	T	**T**	**T**	**F**	**T**	**F**	**F**
T	T	F	**F**	**T**	**F**	**F**	**F**	**F**
T	F	T	**T**	**T**	**F**	**T**	**F**	**T**
T	F	F	**F**	**F**	**T**	**T**	**F**	**T**
F	T	T	**F**	**T**	**F**	**F**	**T**	**T**
F	T	F	**F**	**T**	**F**	**F**	**T**	**T**
F	F	T	**F**	**T**	**F**	**T**	**T**	**T**
F	F	F	**F**	**F**	**T**	**T**	**T**	**T**

1C Sets and Venn Diagrams

3. Draw a Venn diagram for each categorical proposition. Then write on the line whether the sets are disjoint, overlapping, or one is a subset of the other.
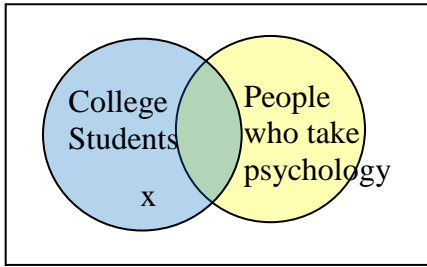
Relationship between the Sets

c. No cars are airplanes.   _____*Disjoint*_____
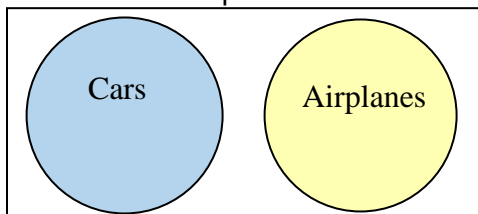
d. Some college students don't take psychology.   _____Overlapping_____

4. Interpreting values on a Venn diagram
A survey was taken to see which professional sports were watched by students. Let F represent football, B represent basketball, and H represent hockey.

Answer the following questions (use complete sentences). Show any calculations that were used.
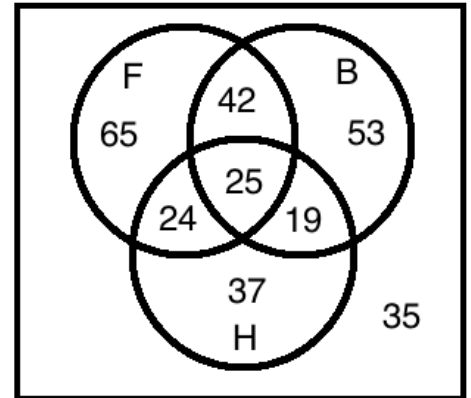
   a. How many students participated in the survey?

       65 + 42 + 53 + 24 + 25 + 19 + 37 + 35 =300
       300 students took the survey.

   b. How many students watched basketball?

       42 + 53 + 25 + 19 = 139
       139 students watched basketball.

   c. How many students watched only one of the sports?

       65 + 53 + 37 = 155
       155 students watched only one of the sports.

   d. How many students watched at least two sports?

       24 + 42 + 19 + 25 = 110
       110 students watched at least two of the three sports.

   e. How many students didn't watch hockey?

       300 – (24 + 25 + 19 + 37) = 195   OR   35 + 65 + 42 + 53 = 195
       195 students didn't watch hockey.

   f. How many students watched football and hockey, but not basketball?

       (this one can be found directly from the diagram)
       24 students watched football and hockey, but not basketball.

1D Analyzing Arguments

5. Draw a Venn diagram for each deductive argument and determine whether it is valid and sound. (Problems adapted from David Lippman http://www.opentextbookstore.com/mathinsociety/index.html)

a.   Premise: Some cows are brown
          Premise: Fido is not a cow
          Conclusion: Fido is brown
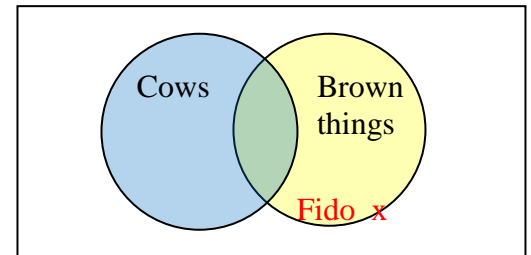
          The conclusion is valid/==invalid==
          The conclusion is sound/==not sound==

          The conclusion does not follow from the premises, so the argument is invalid. Because it is invalid it cannot be sound.

b.   Premise: All PCC Students Live in Portland *(F)*

          Premise: Trung is a PCC Student *(T)*

          Conclusion: Trung lives in Portland *(Unknown)*
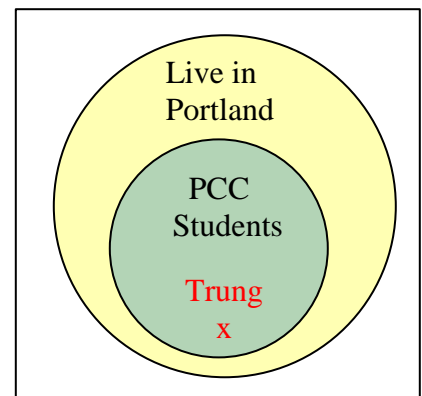
          The conclusion is ==valid==/invalid
          The conclusion is sound/==not sound==

          The conclusion follows from the premises so the argument is valid. The first premise is false, however, so the argument is not sound.

c.   Premise: All firefighters know CPR
          Premise: Juliana knows CPR
          Conclusion: Juliana is a firefighter
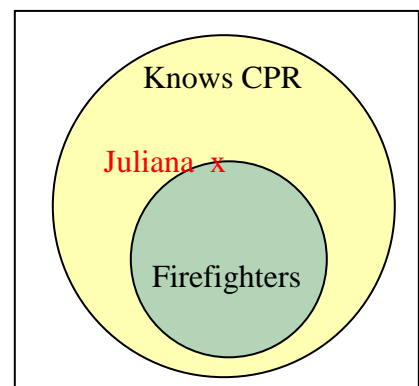
          The conclusion is valid/==invalid==
          The conclusion is sound/==not sound==

          The conclusion does not follow from the premises. It does not matter whether the premises are true or false, the argument is invalid and therefore it is not sound.

6. Here is a conditional statement: If I go to the party, I will see Shonda.

Identify whether each statement is using the original, converse, inverse or contrapositive. Which two of these arguments are valid?

> a. I did not see Shonda; therefore, I did not go to the party.
> > **Not q, then not p. Contrapositive.**

> b. I saw Shonda; therefore, I went to the party.
> > **q, then p. Converse.**

> c. I went to the party; therefore, I saw Shonda.
> > **p, then q. Original Statement.**

> d. I did not go to the party; therefore, I did not see Shonda.
> > **Not p, then not q. Inverse.**

7. Here is a conditional statement: If it is raining, there are clouds in the sky.

Identify whether each statement is using the original, converse, inverse or contrapositive. Which two of these arguments are valid?

> a. It is not raining; therefore, there are no clouds in the sky.
> > **Not p, then not q. Inverse.**

> b. There are no clouds in the sky; therefore, it is not raining.
> > **Not q, then not p. Contrapositive.**

> c. There are clouds in the sky; therefore, it is raining.
> > **q, then p. Converse.**

> d. It is raining; therefore, there are clouds in the sky.
> > **p, then q. Original Statement.**

8. Given your answers to problems 4, 5, and 6, which of the four types of statements are equivalent to each other? Challenge problem: prove it with a truth table.

The original statement and the contrapositive are equivalent to each other.
The inverse and converse are equivalent to each other.

1D Identifying Premise(s) and Conclusions
1A Logical Fallacies

9. The conclusion is typically written after the premise(s), but that is not always the case. In each problem, highlight or circle each premise(s) and label them. Underline the conclusion and label it. Then write the type of fallacy.

premise   *conclusion*

Example:
The poll shows that Mr. Burke is leading by a 2-to-1 margin, so you should vote for Mr. Burke.

Fallacy: **Appeal to Popularity. Just because a candidate is leading in the polls that doesn't mean I should vote for them or that they will win.**

a. Coyotes must be extinct, since I haven't seen any for five years.

Fallacy: **Appeal to Ignorance. Just because I haven't seen any doesn't mean they don't exist.**

b. "You don't drink Coke, so you must not consume caffeinated drinks."

Fallacy: **Limited Choice. The argument assumes that the only choice for a caffeinated drink is Coke. There are obviously many other possibilities.**

c. People should avoid seeing the movie "Star Wars" because Jan was diagnosed with strep throat the next day after she saw it, and I got the flu a few days after seeing that same movie.

Fallacy: **False Cause. This argument is implying that watching the movie caused them to get sick.**

d. "Ms. Cheng is the most successful mayor of Bay City because she is the best mayor in the history of Bay City."

Fallacy: **Circular Reasoning (stating the same thing different ways). The statement basically says that Ms. Cheng is the best mayor because she is the best mayor.**